Name: Feedback

SA367 · Mathematical Models for Decision Making Spring 2022 · Uhan

Quiz 1 – 1/20/2022

Instructions. You have 15 minutes to complete this quiz. You may not use any other materials (e.g., notes, homework, website).

Show all your work. To receive full credit, your solutions must be completely correct, sufficiently justified, and easy to follow.

Problem	Weight	Score
1	3	
2	0.5	
3	0.5	
Total		/ 40

Problem 1. Fluttering Duck Airlines is starting operations at the small airport in Simplexville. The airline needs to purchase a new tractor to bring luggage to and from the airplanes. A new mechanized system will be installed in 3 years, so the tractor will not be needed after that. However, the tractor will receive heavy use, so the running and maintenance costs will increase rapidly after it ages. As a result, it may still be more economical to replace the tractor after 1 or 2 years. The total net cost of purchasing a tractor at the beginning of year i and trading it in at the beginning of year j is (in thousands of $):

$i\downarrow$ $j\rightarrow$	2	3	4
1	8	18	31
2		10	21
3			12

The goal is to determine what times (if any) the tractor should be replaced to minimize the total cost of having a tractor over the next 3 years.

Formulate this problem as a shortest path problem. In particular:

- draw the directed graph (nodes and edges),
- specify the edge lengths, and
- specify the source and sink nodes.

Almost all of you had the right idea here.

In your directed graphs, do not forget to indicate the direction of the edges! This is critical – without the edge directions, it is not clear what constitutes a valid path.

Suppose you solved the shortest path problem you formulated in Problem 1 with an algorithm that outputs (i) the length of a shortest path, and (ii) the nodes and edges in a shortest path.

Problem 2. Briefly explain how you would use this output to determine the minimum total cost of having a tractor over the next 3 years.

See the next page.

Problem 3. Briefly explain how you would use this output to determine when to purchase a new tractor. Give a hypothetical example if it helps.

See the next page.

Most of you had the right idea with Problems 2 and 3.

If you struggled with these problems, here's a suggestion on how to answer them:

- Start by giving hypothetical outputs for the algorithm described: a shortest path (ii) and its length (i).
- Next, based on these hypothetical outputs, state the minimum total cost of having a tractor over the next 3 years (for Problem 2), or when to purchase a new tractor (for Problem 3).
- For Problem 2, you should end up with something like this:

  > Suppose a shortest path from node 1 to node 4 is (1, 3), (3, 4). The corresponding length is 30. In this case, the minimum total cost of having a tractor over the next 3 years is **[fill in the blank]**.

- For Problem 3, you should end up with something like this:

  > Suppose a shortest path from node 1 to node 4 is (1, 3), (3, 4). The corresponding length is 30. In this case, the airline should purchase a new tractor **[fill in the blank]**.

- By following the steps above, you can concisely describe how to translate a shortest path's length and edges into a solution for the original problem.
- Note that the hypothetical shortest path given above is not actually a shortest path. You don't need to find an actual shortest path – we will leave this to the computer. You just need to describe how to translate a shortest path's lengths and edges into a solution for the original problem.
- Take a look at Examples 4-7 in Lesson 1 for examples of how we translated a shortest path's length and edges into solutions for the original problem.